



8.1 The superposition principle





Introduction

- In this topic, we will
 - Describe the superposition principle
 - Give some examples
 - Examine the need for functions that preserve vector space operations





Background

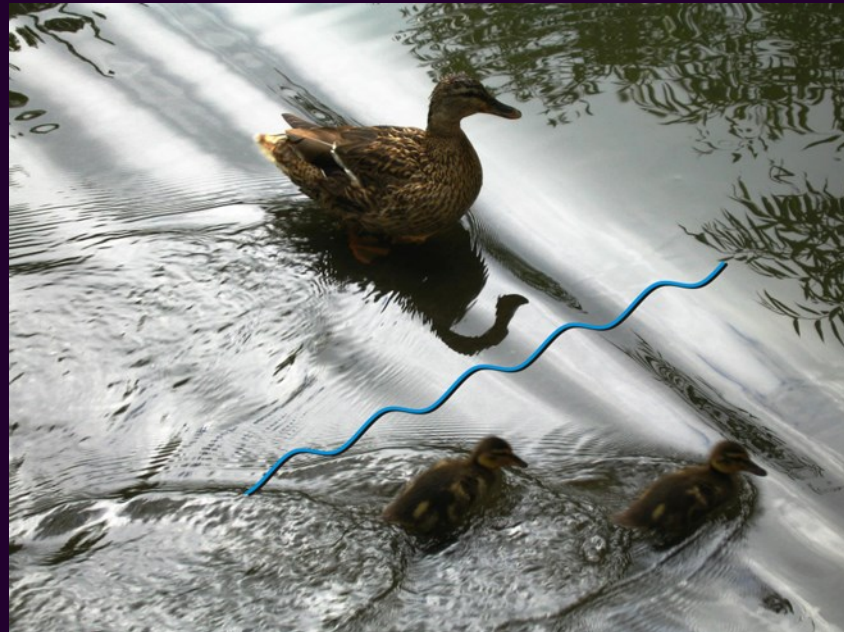
- The *superposition principle* is an observation from physics that there are many systems where the net response at a given place and time caused by two or more stimuli is the sum of the response that would have been caused by each stimulus individually.





The superposition principle

- This image shows the superposition of waves:



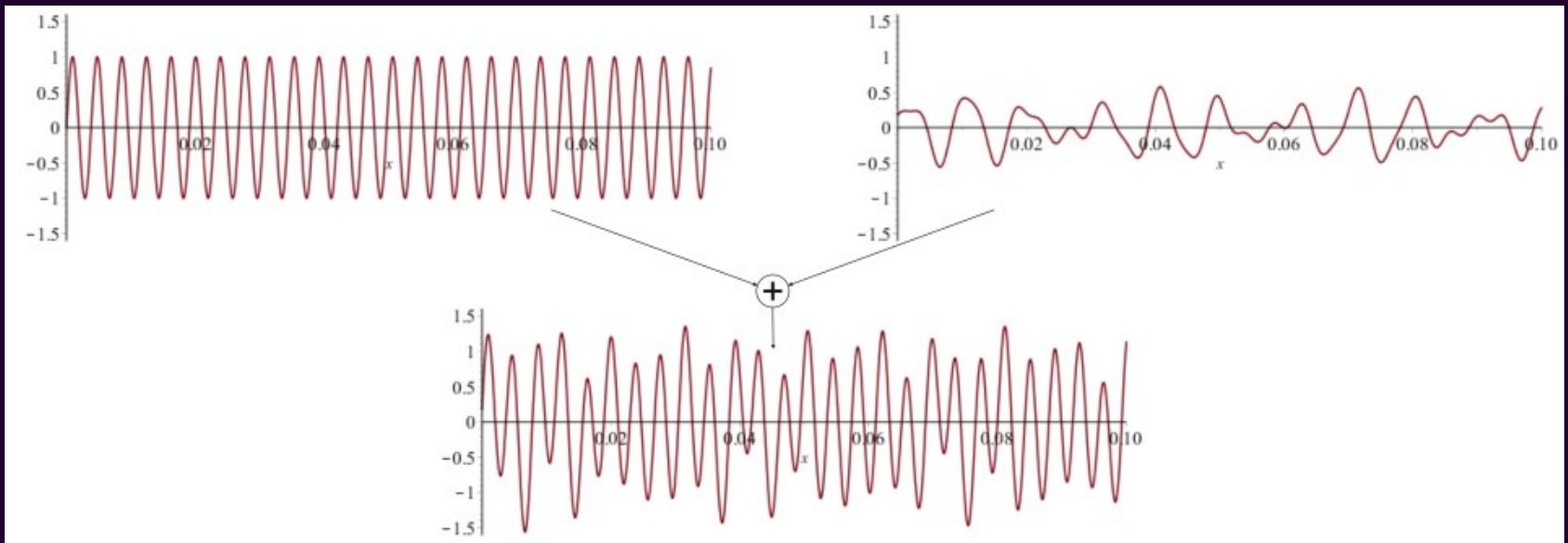
- One specific single wave, a *soliton*, does not obey the superposition principle when two such waves meet





Examples

- Here we see middle C combined with noise



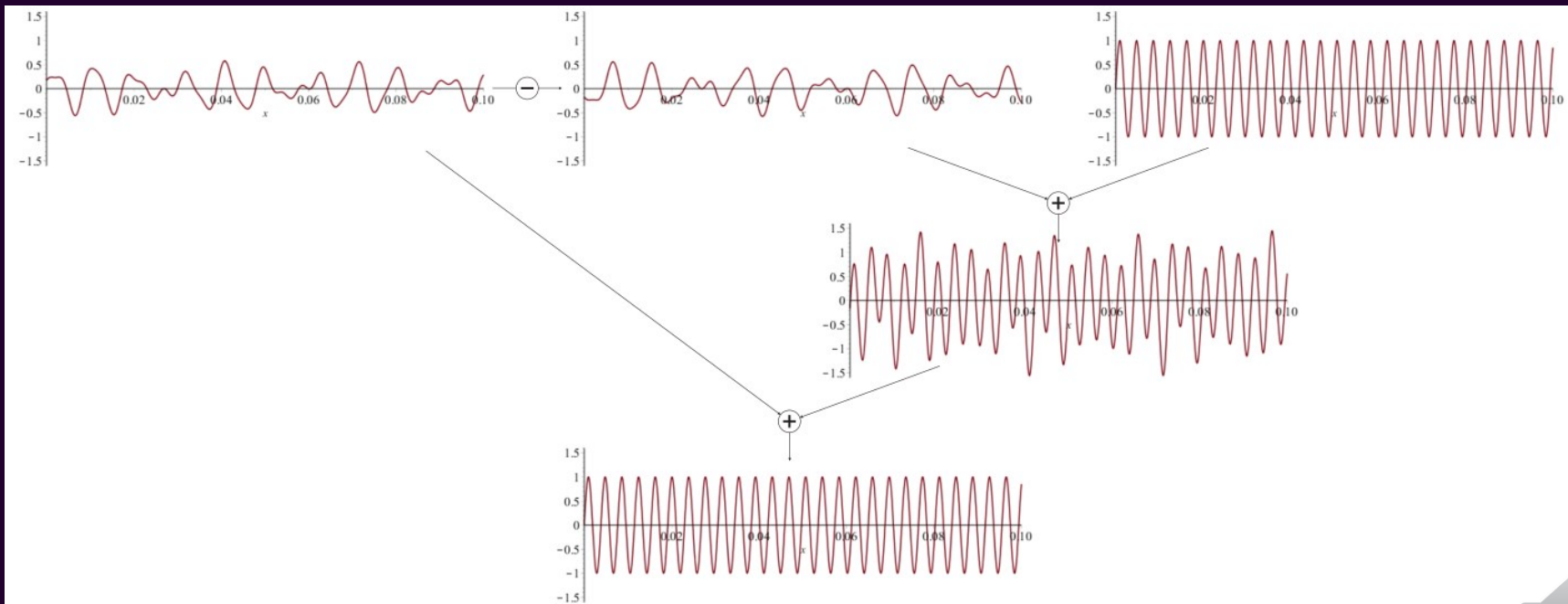
- One goal of sound engineering is to remove noise





Examples

- The goal of noise-cancelling headphones is to:
 - Estimate the noise vector $\mathbf{u}$
 - Introduce $-\mathbf{u}$ to the sound generated by the headphones
 - This cancels out the outside noise





Examples

- The gravitational and electromagnetic forces also obey the superposition principle

$$\begin{aligned} F_{\text{gravitational}} &= m_0 \mathbf{a}_k = \sum_{k=1}^n G \frac{m_0 m_k}{\|\mathbf{x}_0 - \mathbf{x}_k\|_2^3} (\mathbf{x}_0 - \mathbf{x}_k) \\ &= \left(G m_0 \sum_{k=1}^n \frac{m_k}{\|\mathbf{x}_0 - \mathbf{x}_k\|_2^3} \right) \mathbf{x}_0 - G m_0 \sum_{k=1}^n \frac{m_k \mathbf{x}_k}{\|\mathbf{x}_0 - \mathbf{x}_k\|_2^3} \end{aligned}$$

$$\begin{aligned} F_{\text{electric}} &= m_0 \mathbf{a}_k = \sum_{k=1}^n k_e \frac{q_0 q_k}{\|\mathbf{x}_0 - \mathbf{x}_k\|_2^3} (\mathbf{x}_0 - \mathbf{x}_k) \\ &= \left(k_e q_0 \sum_{k=1}^n \frac{q_k}{\|\mathbf{x}_0 - \mathbf{x}_k\|_2^3} \right) \mathbf{x}_0 - k_e q_0 \sum_{k=1}^n \frac{q_k \mathbf{x}_k}{\|\mathbf{x}_0 - \mathbf{x}_k\|_2^3} \end{aligned}$$





Examples

- Also, alternating current (AC) circuits with resistors, inductors and capacitors display a vector-like behavior:
 - If you double a current, the voltage response double
 - Scalar multiplication
 - If you add two currents, the voltage response is the sum of the individual voltage response
 - Vector addition





Consequence

- Engineers prefer to work with mappings that preserve vector space operations
 - The response to a sum of inputs,
should be the sum of the responses to the individual inputs
 - The response to an *attenuated* or *amplified* input,
should be the response similarly attenuated or amplified
- For example, if you develop a sound system that
 - Nearly eliminates white noise
 - Leaves the volumes of human voices essentially unchangedthen when you mix white noise with human voices,
the noise should still be eliminated while the human voices remain





Looking forward

- Consequently, we will look at linear maps and linear operators:

$$f(\alpha \mathbf{u} + \beta \mathbf{v}) = \alpha f(\mathbf{u}) + \beta f(\mathbf{v})$$

- You have already seen one linear operator
the derivative:

$$\frac{d}{dx}(\alpha f(x) + \beta g(x)) = \alpha \frac{d}{dx} f(x) + \beta \frac{d}{dx} g(x)$$





Summary

- Following this topic, you now
 - Know that the superposition principle comes from natural phenomena
 - Understand these phenomena can be described through vector space operations
 - Can see the potential need for functions that preserve vector space operations
 - The response to a linear combination of inputs, is that same linear combination of the individual responses





References

- [1] https://en.wikipedia.org/wiki/Superposition_principle





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas. Mathematical equations are prepared in MathType by Design Science, Inc. Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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